CS 6110 – Formal Methods in System Design | Spring 2015
Apr-1

Lecture 14
# SMT Solvers

Zvonimir Rakamarić
University of Utah

slides acknowledgements: Leonardo de Moura

## This Time

- SMT solvers
  - What are they?
  - How they work?

## Many Theories

- Theory of equality
- Peano arithmetic
- Presburger arithmetic
- Linear integer arithmetic
- Reals
- Rationals
- Arrays
- Recursive data structures
- …

## Combination of Theories

- In practice, we often need a combination of theories
- Example:
  $x+2=y \rightarrow f(select(store(a,x,3),y-2)=f(y-x+1)$
- Problem: given satisfiability procedures for conjunction of literals of $\text{Theory}_1$ and $\text{Theory}_2$, how to decide satisfiability of their combination?

## Satisfiability Modulo Theories (SMT) Solver

- Satisfiability checker with built-in support for useful theories
  - Arithmetic
  - Equality with uninterpreted functions
  - Arrays
  - …
- Combines a SAT solver with theory solvers
- Next generation of reasoning engines
  - Automatic
  - Fast

## SMT Solvers, Library, Competition

- Solvers
  - AProve, Barcelogic, Boolector, CVC4, MathSAT5, OpenSMT, SMTInterpol, SOLONAR, STP2, veriT, Yices, Z3
- SMT-LIB
  - Standardizes various theories and input format
  - Library of benchmarks
  - http://www.smtlib.org
- SMT-COMP
  - Annual competition
  - http://www.smtcomp.org

## Applications

- Test case generation
- Verifying compilers
- Software verification
- Hardware verification
- Equivalence checking
- Type checking
- Model based testing
- Scheduling and planning
- …

## Nelson-Oppen Combination Procedure

- Initial State
  - F is a conjunction of literals over $\Sigma_1 \cup \Sigma_2$
- Purification
  - Preserving satisfiability transform F into $F_1 \wedge F_2$, such that $F_i \in \Sigma_i$
- Interaction
  - Deduce an equality x = y if $F_1 \rightarrow x = y$, where x and y are common (shared) variables
  - Update $F_2 := F_2 \wedge x = y$
  - And vice-versa
  - Repeat until no further changes

## Nelson-Oppen Combination Procedure

- Component procedures
  - Use individual decision procedures to decide whether $F_i$ is satisfiable
- Return
  - If both return yes, return yes
  - No, otherwise

- Remark:
  $F_i \rightarrow x = y$ iff $F_i \wedge x \neq y$ is not satisfiable

## Purification Example

$f(x - 1) - 1 = x \wedge f(y) + 1 = y$

## Nelson-Oppen Procedure Example I

$x + y = z \wedge f(z) = z \wedge f(x + y) \neq z$

## Nelson-Oppen Procedure Example II

$x+2=y \wedge f(select(store(a,x,3), y - 2)) \neq f(y - x + 1)$

## Building an Efficient Solver

## Eager Approach

- Translate formula into equisatisfiable propositional formula and use off-the-shelf SAT solver
- Why “eager”?
  - Search uses all theory information from the beginning
- Can use best available SAT solver
- Sophisticated encodings are need for each theory
- Sometimes translation and/or solving too slow

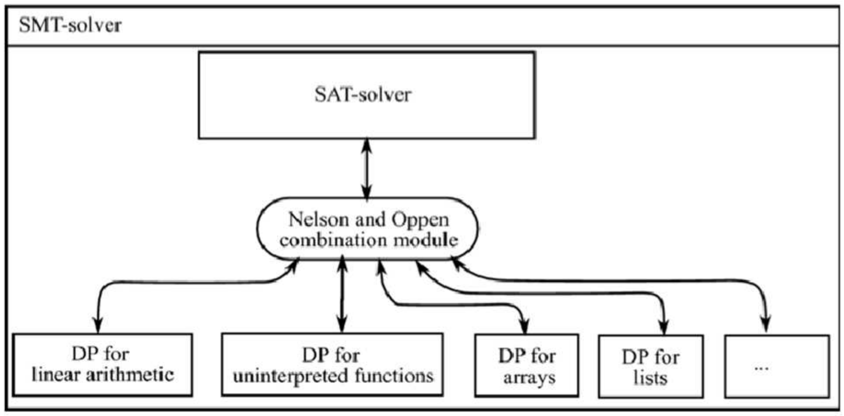
## Lazy Approach: SAT + Theories I

- Independently developed by several groups
  - CVC (Stanford)
  - ICS (SRI)
  - MathSAT (Univ. Trento, Italy)
  - Verifun (HP)
- Motivated by the breakthroughs in SAT solving
  - DPLL algorithm
  - Various optimizations and heuristics

## Lazy Approach: SAT + Theories II

- SAT solver
  - Manages the boolean structure and assigns truth values to the atoms in a formula
- Theory solvers
  - Efficiently validate (partial) assignments produced by the SAT solver
- When a theory solver detects unsatisfiability, a new clause (lemma) is created

## Basic architecture




## Naïve Approach

- Example
  - Suppose SAT solver assigns {x = y → T, y = z → T, *f*(x) = *f*(z) → F}
  - Theory solver detects conflict
  - Lemma is created ¬(x = y) ∨ ¬(y = z) ∨ *f*(x) = *f*(z)
- Potential problems
  - Lemmas are imprecise (not minimal)
  - Theory solver is “passive”
    - It just detects conflicts
    - There is no propagation step
  - Backtracking is expensive
    - Restart from scratch when a conflict is detected

## Theory Solvers

- Basic requirements
  - Deduce equalities between variables
  - Compute lemmas (conflict sets)
    - As precise as possible
- Extra desired features
  - Theory propagation
  - Incrementality
  - Backtracking

## Equality Generation

- Combination of theories strongly relies on the propagation of deduced equalities
- Every theory solver has to support it

## Precise Lemmas I

- Example
  - $\{a_1 = T, a_2 = F, a_3 = F\}$ is inconsistent
  - Lemma is $\neg a_1 \vee a_2 \vee a_3$
- An inconsistent set $A$ is redundant if $A' \subset A$ is also inconsistent
- Redundant inconsistent sets imply
  - Imprecise lemmas
  - Ineffective pruning of the search space

## Precise Lemmas II

- Noise of a redundant set is $A \setminus A_{min}$
- Imprecise lemma is useless in any partial assignment where an atom in the noise has a different assignment
- Example
  - Suppose $a_1$ is in the noise
  - Then $\neg a_1 \vee a_2 \vee a_3$ is useless when $a_1 = F$

## Theory Propagation

- SAT solver is assigning truth values to the atoms in a formula
- Partial assignment produced by the SAT solver may imply truth values of unassigned atoms
- Example

  $x = y \wedge y = z \wedge (f(x) \neq f(z) \vee f(x) = f(w))$

  Partial assignment $\{x = y \rightarrow T, y = z \rightarrow T\}$ implies $f(x) = f(z)$
- Reduces the number of conflicts and the search space

## Incrementality

- Theory solvers constantly receive new constraints and restart the process
  - Augmented partial assignments from SAT solver
  - Equalities coming from other theory solvers
- Do not restart from scratch
  - Reuse what you learned so far

## Efficient Backtracking

- One of the most important improvements in SAT was efficient backtracking
- Extreme (inefficient) approach in theory solvers
  - Restart from scratch on every conflict
- Efficient approach
  - Restore to a logically equivalent state
- Backtracking should be included in the design of theory solvers

## Ideal Theory Solver

- Efficient in real benchmarks
- Produces precise lemmas
- Supports theory propagation
- Incremental
- Efficient backtracking