Lecture 3
SAT

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Last Time

- Propositional logic
  - Syntax
  - Semantics
- Satisfiability and validity
- Normal forms
  - Negation normal form (NNF)
  - Disjunctive Normal Form (DNF)
  - Conjunctive Normal Form (CNF)
    - Input to SAT

This Time

- Boolean satisfiability problem or SAT

Decision Procedure for Satisfiability

- Algorithm that in some finite amount of computation decides if given PL formula *F* is satisfiable
  - NP-complete problem
- Modern decision procedures for PL formulae are called *SAT solvers*
- Naïve approach
  - Enumerate truth table
- Modern SAT solvers
  - DPLL algorithm
    - Davis-Putnam-Logemann-Loveland
  - Operates on Conjunctive Normal Form (CNF)

Classical DPLL

- Searching for a model *M* for a given CNF formula *F*
  - Incrementally try to build a model *M*
  - Maintain state during search
- State is a pair *M* | *F*
  - *F* is a set of clauses and it doesn't change during search
  - *M* is a sequence of literals
    - No literals appear twice and no contradiction
    - Order does matter
    - Decision literals marked with l^d

Abstract Transition System

- Contains a set of rules of the form

$$M \mid F \Rightarrow M' \mid F'$$

  denoting that search can move from state $M \mid F$ to state $M' \mid F'$

DPLL Rules – Extending M

- Propagate

$M \mid G, C \vee l \Rightarrow M, l \mid G, C \vee l$
if $M \models \neg C$ and l not in M

- Decide

$M \mid F \Rightarrow M, l^d \mid F$
if l or $\neg l$ in F and l not in M

DPLL Rules – Adjusting *M*

- Fail

$M \mid G,C \Rightarrow \mathit{fail}$
if $M \models \neg C$ and M contains no decision literals

- Backtrack

$M,l^d,N \mid G,C \Rightarrow M,\neg l \mid G,C$
if $M,l^d,N \models \neg C$ and N contains no decision literals

- Propagate

$M \mid G, C \vee l \Rightarrow M, l \mid G, C \vee l$
if $M \models \neg C$ and l not in M

- Decide

$M \mid F \Rightarrow M, l^d \mid F$
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- Fail

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- Backtrack

$M, l^d, N \mid G, C \Rightarrow M, \neg l \mid G, C$
if $M, l^d, N \models \neg C$ and N contains no decision literals

DPLL Example 1

$\emptyset$ | $\neg p \vee q \vee r$, p, $\neg q \vee r$, $\neg q \vee \neg r$, $q \vee r$, $q \vee \neg r$

DPLL Example 2

$\emptyset$ | $\neg p \vee q$, $\neg r \vee s$, $\neg t \vee \neg u$, $u \vee \neg t \vee \neg q$

Modern SAT Solvers

- DPLL + improvements
  - Backjumping
  - Dynamic variable ordering
  - Learning conflict clauses
  - Random restarts
  - …

Next Time

- More on boolean satisfiability
  - Using a SAT solver
  - Encoding a problem into SAT