

**Lecture 19**

# **Course Summary**

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# Announcements

- ▶ Project presentations on Wednesday in class
  - ▶ Don't go over your time limit
- ▶ Final project report due on Apr 29
  - ▶ Can be pushed back if you email me

# Propositional Logic

- ▶ Syntax
- ▶ Semantics
- ▶ Normal forms
  - ▶ Negation Normal Form (NNF)
  - ▶ Disjunctive Normal Form (DNF)
  - ▶ Conjunctive Normal Form (CNF)
- ▶ Tseitin transformation for generating CNF

# SAT

- ▶ Algorithm that in some finite amount of computation decides if given propositional logic formula is satisfiable
- ▶ Modern SAT solvers
  - ▶ DPLL algorithm
  - ▶ Operates on Conjunctive Normal Form (CNF)
- ▶ Using MiniSAT
- ▶ Encoding problems into SAT

# First-Order Logic

- ▶ Extends propositional logic with predicates, functions, and quantifiers
- ▶ Syntax
- ▶ Semantics
- ▶ Satisfiability and validity
- ▶ Proving validity with semantic argument method using proof rules

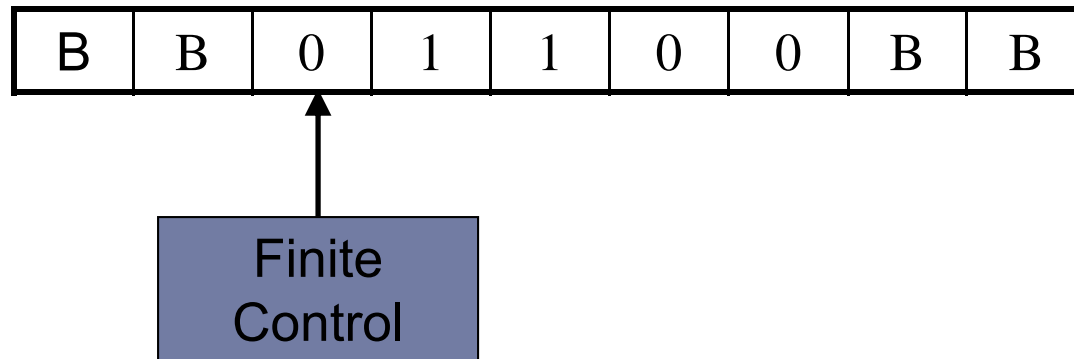
# First-Order Theories

- ▶ Introduced theories
  - ▶ Theory of equality
  - ▶ Peano arithmetic
  - ▶ Presburger arithmetic
  - ▶ Linear integer arithmetic
  - ▶ Reals and Rationals
  - ▶ Arrays
- ▶ Decidability and complexity
- ▶ Z3 SMT solver

# SMT Solvers

- ▶ Nelson-Oppen algorithm for satisfiability of a combination of first-order theories
- ▶ Modern SMT solvers combine SAT with Nelson-Oppen
  - ▶ SAT solver
    - ▶ Manages the boolean structure and assigns truth values to the atoms in a formula
  - ▶ Theory solvers
    - ▶ Efficiently validate (partial) assignments produced by the SAT solver
- ▶ Encoding problems into SMT

# Turing Machine and Computability



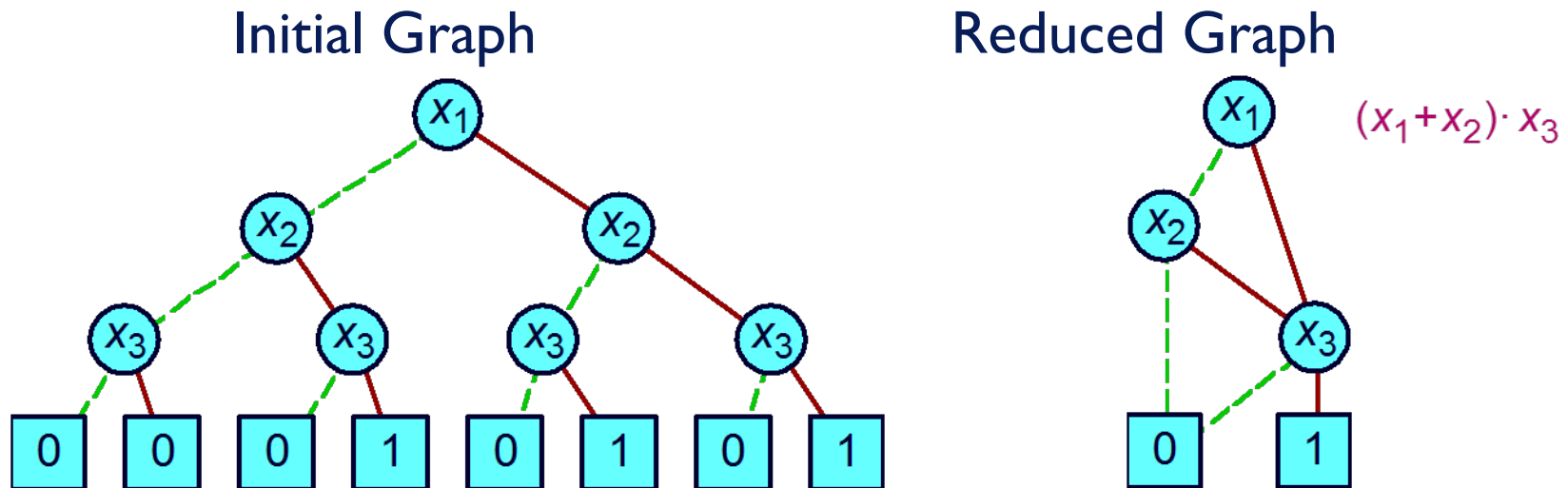
- ▶ Any mathematical problem solving that can be described by a mechanical procedure (algorithm) can be modeled by a Turing machine.
- ▶ Halting problem
  - ▶ Proof of undecidability
- ▶ Rice's theorem
  - ▶ Any interesting question about the behavior of a program is undecidable



# Complexity

- ▶ Decision problems
- ▶ P complexity class
  - ▶ Poly-time algorithm
- ▶ NP complexity class
  - ▶ Poly-time certifier
- ▶ Polynomial transformation
- ▶ NP-complete problems
  - ▶ A problem in NP with the property that every other problem in NP can be polynomially transformed into it
- ▶ Proving NP-completeness

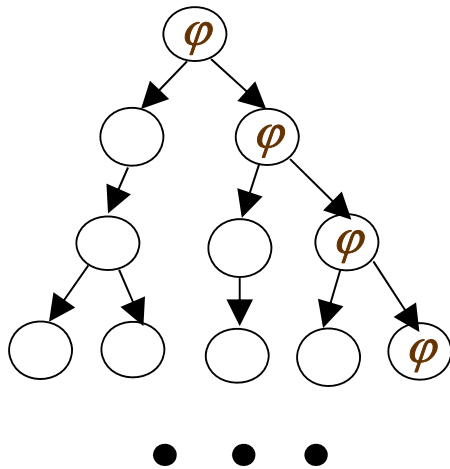
# Binary Decision Diagrams



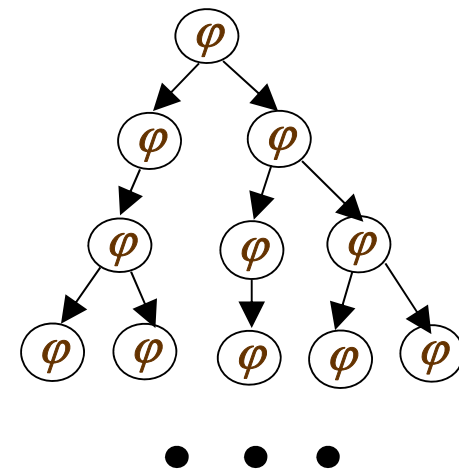
- ▶ Canonical data structure for representing quantifier-free boolean formulas
- ▶ Efficient boolean operations

# Model Checking

- ▶ Good for finding bugs in hardware and software
- ▶ Kripke structures
- ▶ Temporal logics
  - ▶ Computational tree logic (CTL)



**EG (exists global)**



**AG (all global)**

# Model Checking cont.

- ▶ Explicit state algorithm for CTL model checking
  - ▶ Recursively label states with CTL formulas that hold in a state
- ▶ Using NuSMV
- ▶ Symbolic model checking
  - ▶ Sets of states and the transition relation are represented implicitly by formulas
  - ▶ Set operations are defined in terms of formula manipulations
  - ▶ Use BDDs

# Next Time

- ▶ Student project presentations