Lecture 6

Verification Condition Generator

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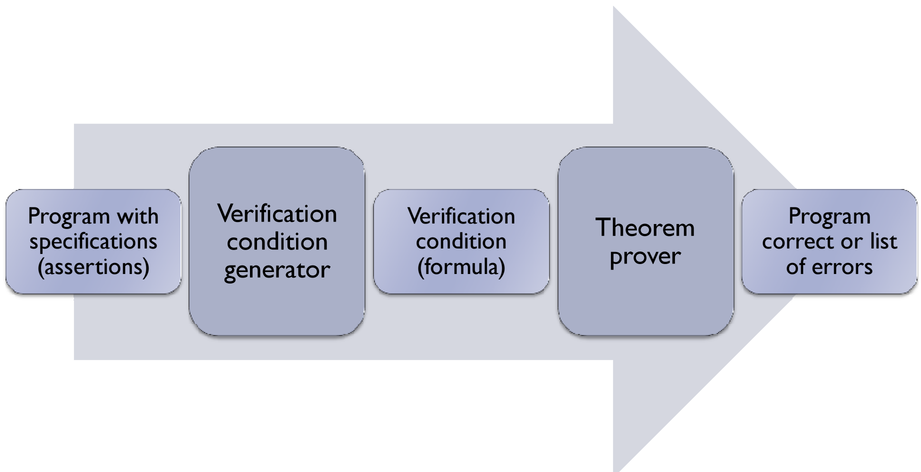
This Time

- Homework 2 questions?
- Verification condition generation
- Weakest precondition transformer

Basic Verifier Architecture



Verification Condition Generator

- Creates verification conditions (mathematical logic formulas) from program's source code
  - If VC is valid – program is correct
  - If VC is invalid – possible error in program
- Based on the theory of Hoare triples
  - Formalization of software semantics for verification
- Verification conditions computed automatically using *weakest preconditions* (wp)

Simple Command Language

x := E

havoc x

assert P

assume P

S ; T [sequential composition]

S T [choice statement]

Program States

- Program state *s*
  - Assignment of values (of proper type) to all program variables
  - Sometimes includes program counter variable *pc*
    - Holds current program location
- Example

  $s : (x \mapsto -1, y \mapsto 1)$

  $s : (pc \mapsto L, a \mapsto 0, i \mapsto 3)$
- Reachable state is a state that can be reached during some computation

Program States cont.

- A set of program states can be described using a FOL formula
- Example

  Set of states:

  $s : \{ (x \mapsto 1), (x \mapsto 2), (x \mapsto 3) \}$

  FOL formulas defining s:

  $x = 1 \lor x = 2 \lor x = 3$

  $0 < x \land x < 4$ [if x is integer]

Hoare Triple

- Used for reasoning about (program) executions

$$\{ P \} \; S \; \{ Q \}$$

- S is a command
- P is a precondition – formula about program state before S executes
- Q is a postcondition – formula about program state after S executes

Hoare Triple Definition

$$\{ P \} \; S \; \{ Q \}$$

- When a state *s* satisfies precondition P, every terminating execution of command S starting in *s*
  - does not go wrong, and
  - establishes postcondition Q

Hoare Triple Examples

- {a = 2} b := a + 3; {b > 0}
- {a = 2} b := a + 3; {b = 5}
- {a > 3} b := a + 3; {a > 0}
- {a = 2} b := a * a; {b > 0}

Weakest Precondition

- The most general (i.e., weakest) P that satisfies

  { P } S { Q }

  is called the weakest precondition of S with respect to Q, written:

  wp(S, Q)

- To check { P } S { Q } prove $P \rightarrow wp(S, Q)$
- Example

  {?P?} b := a + 3; {b > 0}

  {a + 3 > 0} b := a + 3; {b > 0}

  wp(b := a + 3, b > 0) = a + 3 > 0

Weakest Preconditions Cookbook

- wp(x := E, Q) = Q[E / x]
- wp(havoc x, Q) = ($\forall$ x . Q)
- wp(assert P, Q) = $P \wedge Q$
- wp(assume P, Q) = $P \rightarrow Q$
- wp(S ; T, Q) = wp(S, wp(T, Q))
- wp(S T, Q) = wp(S, Q) $\wedge$ wp(T, Q)

Checking Correctness with wp

{true}

```
x := 1;

y := x + 2;

assert y = 3;
```

{true}

Checking Correctness with wp cont.

{true}

wp(x := 1, x + 2 = 3) = $1 + 2 = 3 \wedge$ true

```
x := 1;
```

wp(y := x + 2, y = 3) = $x + 2 = 3 \wedge$ true

```
y := x + 2;
```

wp(assert y = 3, true) = $y = 3 \wedge$ true

```
assert y = 3;
```

{true}

Check: true $\rightarrow 1 + 2 = 3 \wedge$ true

Example II

{x > 1}

```
y := x + 2;
```

```
assert y > 3;
```

{true}

Example II cont.

{x > 1}

wp(y := x + 2, y > 3) = x + 2 > 3

```
y := x + 2;
```

wp(assert y > 3, true) = y > 3 $\wedge$ true = y > 3

```
assert y > 3;
```

{true}

Check: x > 1 $\rightarrow$ (x + 2 > 3)

Example III

{true}

```
assume x > 1;

y := x * 2;

z := x + 2;

assert y > z;
```

{true}

Example III cont.

{true}

wp(assume x > 1, x * 2 > x + 2) = $x>1 \rightarrow x*2 > x+2$

```
assume x > 1;
```

wp(y := x * 2, y > x + 2) = x * 2 > x + 2

```
y := x * 2;
```

wp(z := x + 2, y > z) = y > x + 2

```
z := x + 2;
```

wp(assert y > z, true) = $y > z \wedge true$ = y > z

```
assert y > z;
```

{true}

Structured if Statement

- Just a “syntactic sugar”:
  if E then S else T
  gets desugared into
  (assume E ; S) ▯ (assume ¬E ; T)

Absolute Value Example

```
if (x >= 0) {
  abs_x := x;
} else {
  abs_x := -x;
}
assert abs_x >= 0;
```

Next Time

- Procedures
- Loops
- Loop invariants