Lecture 9

Program Correctness: Strategy

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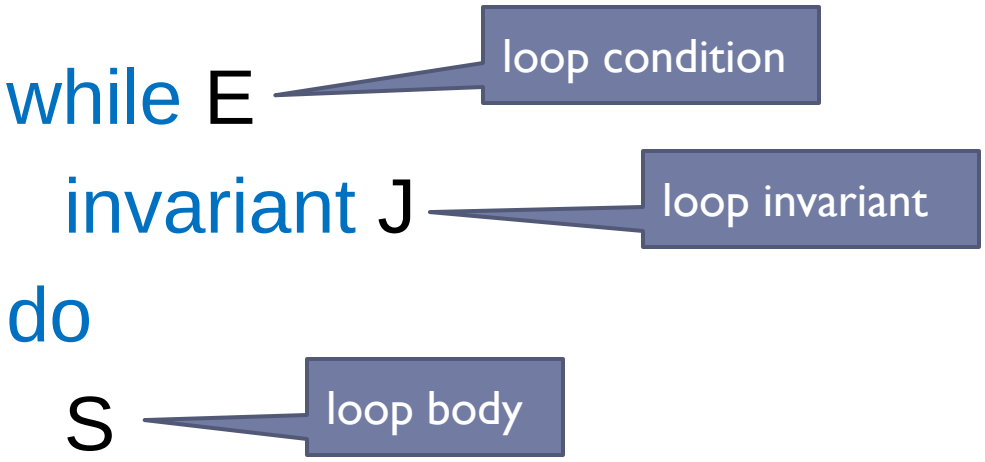
Announcements

- Project proposals due at midnight
  - References (and only references) can be on page 2

Last Time

- Loops
- Loop Invariants
- PiVC verifying compiler with examples

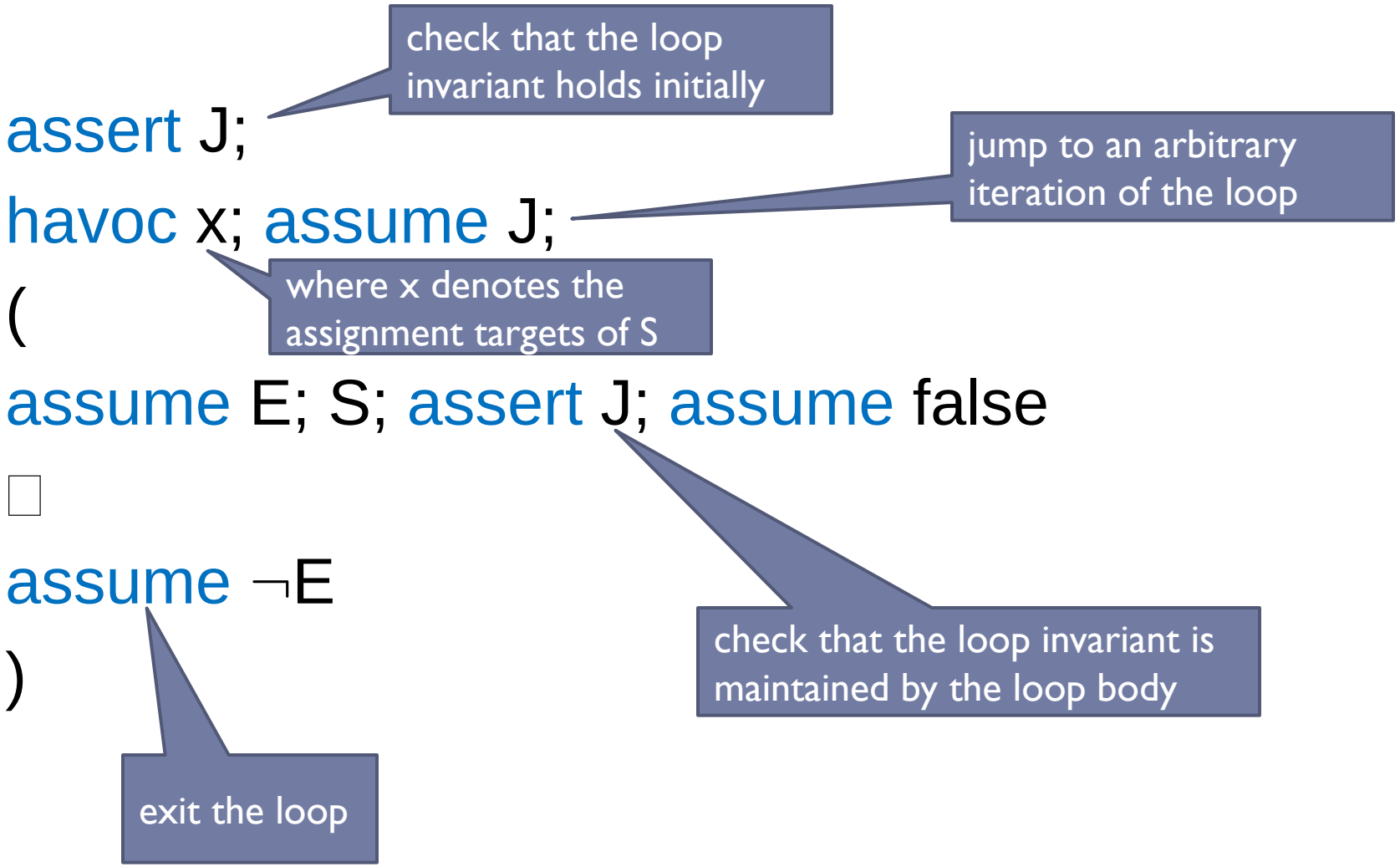
While Loop with Invariant



- Loop body S executed as long as loop condition E holds
- Loop invariant J must hold on every iteration
  - J must hold initially and is evaluated before E
  - J must hold even on final iteration when E is false
  - Provided by a user or inferred automatically

Desugaring While Loop Using Invariant

- while E invariant J do S end



This Time

- Examples, examples, examples…
- Some strategies for proving correctness

Example: Swap

- Signature:

```
int[] swap(int[] a, int i, int j)
```

- Specification:
  - Swaps elements of array `a` at indices `i` and `j` and returns the updated array
- Note:
  - No need for loops, arrays in Pi are copied on assignment

While Loop in Pi

```
i := 0;
while
   @L: 0 <= i && i <= n
   (i < n)
{
   a[i] := i;
   i := i + 1;
}
```

For Loop in Pi

```
for
   @L: 0 <= i && i <= n
   (int i := 0; i < n; i := i + 1)
{
   a[i] := i;
}
```

Quantifiers in Pi

```
exists x. (0 <= x && x <= 10 &&
  a[x] = 0)

forall x. (0 <= x && x <= 10 ->
  a[x] = 0)
```

(Dumb) Example: Multiply by 2

```
int Multiply2(int n) {
 int r := 0;
 for
  (int i := 0; i < n; i := i + 1)
 {
  r := r + 2;
 }
 return r;
}
```

- Specification:
  - Given a non-negative integer `n`, function `Multiply2` multiplies it by 2

Example: Initialize Array

- Signature:

```
int[] InitializeArray(int[] a, int e)
```

- Specification:
  - Initializes elements of array `a` to `e` and returns the updated array

Example: Linear Search I

- Signature:

  ```
  bool LinearSearch(int[] a, int e)
  ```

- Specification:
  - Returns `true` if `e` is found in array `a`, otherwise returns `false`

Example: Linear Search II

- Signature:

```
bool LinearSearch(int[] a, int l,
    int u, int e)
```

- Specification:
  - Returns `true` if `e` is found in array `a` between `l` and `u`, otherwise returns `false`

Binary Search

```
bool BinarySearch(int[] a, int l, int u, int e) {
  if (l > u)
    return false;
  else {
    int m := (l + u) div 2;
    if (a[m] = e)
      return true;
    else if (a[m] < e)
      return BinarySearch(a, m + 1, u, e);
    else
      return BinarySearch(a, l, m - 1, e);
  }
}
```

Bubble Sort

```
@pre  true
@post sorted(rv, 0, |rv| - 1)
int[] BubbleSort(int[] a_0) {
  int[] a := a_0;
  for
    @ true
    (int i := |a| - 1; i > 0; i := i - 1)
  {
    for
      @ true
      (int j := 0; j < i; j := j + 1)
    {
      if (a[j] > a[j + 1]) {
        int t := a[j];
        a[j] := a[j + 1];
        a[j + 1] := t;
      }
    }
  }
  return a;
}
```